A more fundamental International System of Units

David B. Newell

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NIST

The universally accepted method of expressing physical measurements for world commerce, industry, and science is about to get a facelift, thanks to our improved knowledge of fundamental constants.

Although the present International System of Units (SI, from the French Système International d'Unités) was officially established in 1960, its origin goes back to the creation of the metric system during the French Revolution. Following an idea proposed a century earlier by John Wilkins,[1] the new system of weights and measures took as its starting point a single universal measure—the meter—and used it to define length, volume, and mass. The meter came from a perceived constant of nature: one ten-millionth of the distance along Earth's meridian through Paris from the North Pole to the equator.[2] Definitions for the units of volume and mass followed, with the liter being 0.001 m^3 and the kilogram the mass of 1 liter of distilled water at 4 °C. Subsequently, in 1799, two platinum artifact standards for length and mass based on those definitions were deposited in the Archives de la République in Paris. In the words of the Marquis de Condorcet, a new system of measurement "for all time, for all people" was born.

Seventy-six years later, the signing of the Meter Convention in 1875 established three international organizations: the General Conference on Weights and Measures (CGPM), the International Committee for Weights and Measures (CIPM), and the International Bureau of Weights and Measures (BIPM). They were formally tasked with maintaining the SI and continue to do so.

The SI is a living, evolving system, changing as new knowledge and measurement needs arise, albeit sometimes slowly when measured against the rapid pace of scientific progress. For example, in the 18th and 19th centuries when natural philosophers and scientists tried to apply the system of length, mass, and time—with time defined by astronomical observations—to quantify newly discovered phenomena such as magnetism and electricity and the concept of energy, they also discovered the need for

David Newell is a physicist at the National Institute of Standards and Technology in Gaithersburg, Maryland, and chair of the CODATA Task Group on Fundamental Constants.

new units of measure. The likes of Carl Friedrich Gauss, Wilhelm Weber, James Clerk Maxwell, and Lord Kelvin, pioneers in the new science, helped to expand the system and developed the conceptual framework of a coherent system with base mechanical units from which to create derived units as needed. The system included clear explanations of how to realize the base units through measurement, and the coherent derived units were products of powers of the base units with a prefactor of 1.

The timeline in figure 1 shows that despite numerous changes, the SI still has this fundamental framework, with 7 base units (and associated definitions for realizing them) and 22 derived units with special names and symbols.[3] However, international consensus is building to once again advance the SI to reflect contemporary understanding of the physical world. The new framework of the future SI will no longer define seven base units and coherently derived units; instead, it will adopt exact values for seven fundamental constants of nature on which all SI units will be realized. Gone are the base units and their definitions.

How to make a system of units

A system of units to express all physical measurements must take into consideration all physical quantities and the equations that relate those quantities—namely, the accepted laws of physics. A simple example is

$$F = ma = m\, dv/dt = m\, d^2x/dt^2, \qquad (1)$$

where force F, mass m, acceleration a, velocity v, length x, and time t are all quantities and the relations are Newton's second law of motion and basic dynamics.

Carefully choosing a subset of independent base quantities allows one to derive the remaining quantities as functions of the chosen subset through the accepted laws of physics. The selection of base quantities is not unique; but they must be complete and nonredundant.[4] For example, if equation 1 were all we knew about the physical world (six quantities, three constraints), choosing either force or mass and any two of the remaining five quantities would give us an independent set of three base quantities.

However, we are not yet done. To fully define the system of units, we must assign a specific reference quantity to each base quantity. The reference quantity can be a specific artifact, as is the case for the base quantity of mass in the present SI—the international prototype of the kilogram (IPK). Alternatively, in the energy equivalence relations
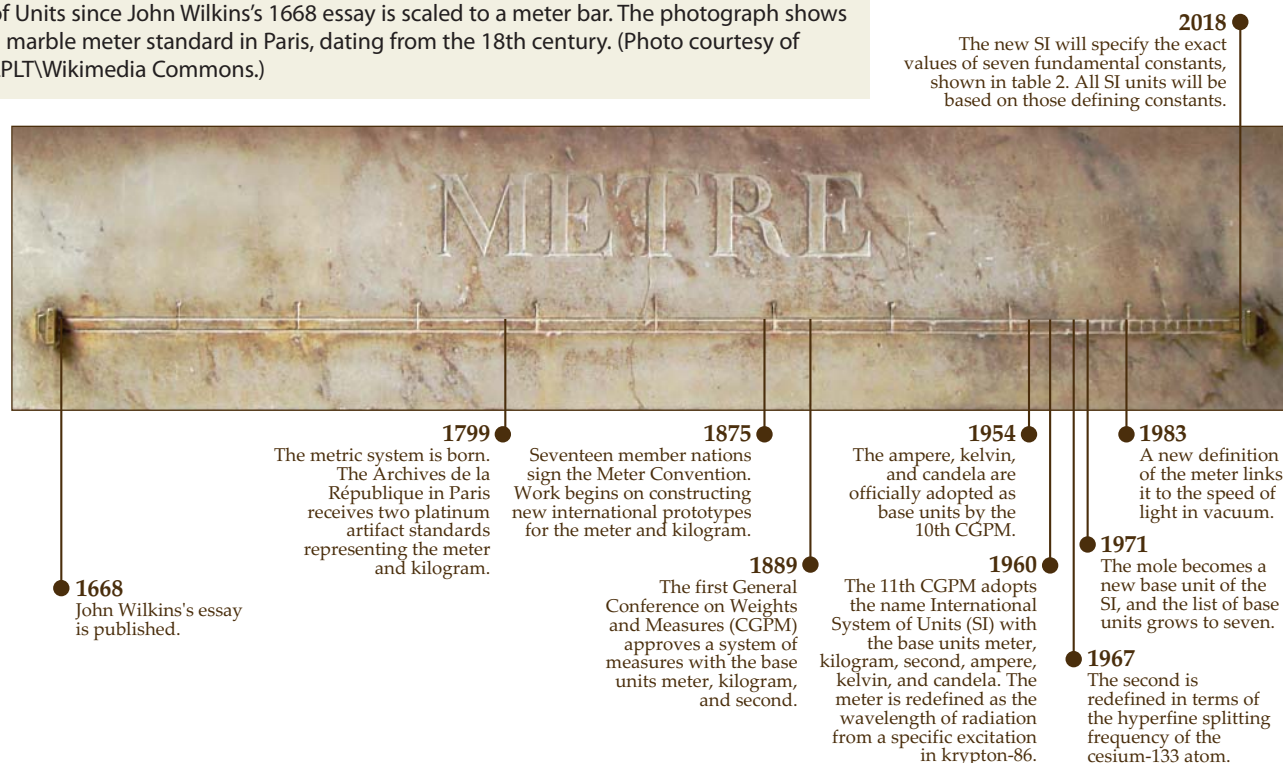
$$E = h\nu = mc^2 = eV = kT, \qquad (2)$$

the Planck constant h, the speed of light c, the elementary charge e, and the Boltzmann constant k can also be reference quantities since they are invariants with specific values.

The present SI has seven base quantities: time, length, mass, electric current, thermodynamic temperature, amount of substance, and luminous intensity. The specific reference quantities are the definitions shown in table 1. In other words, the reference quantities in the present SI are the definitions of the base units: the second, meter, kilogram, ampere, kelvin, mole, and candela.

The new SI will also have seven base quantities:

Figure 1. Evolution of the SI. A brief timeline of the history of the International System of Units since John Wilkins's 1668 essay is scaled to a meter bar. The photograph shows a marble meter standard in Paris, dating from the 18th century. (Photo courtesy of LPLT\Wikimedia Commons.)

2018
The new SI will specify the exact values of seven fundamental constants, shown in table 2. All SI units will be based on those defining constants.

1799
The metric system is born. The Archives de la République in Paris receives two platinum artifact standards representing the meter and kilogram.

1875
Seventeen member nations sign the Meter Convention. Work begins on constructing new international prototypes for the meter and kilogram.

1954
The ampere, kelvin, and candela are officially adopted as base units by the 10th CGPM.

1983
A new definition of the meter links it to the speed of light in vacuum.

1668
John Wilkins's essay is published.

1889
The first General Conference on Weights and Measures (CGPM) approves a system of measures with the base units meter, kilogram, and second.

1960
The 11th CGPM adopts the name International System of Units (SI) with the base units meter, kilogram, second, ampere, kelvin, and candela. The meter is redefined as the wavelength of radiation from a specific excitation in krypton-86.

1971
The mole becomes a new base unit of the SI, and the list of base units grows to seven.

1967
The second is redefined in terms of the hyperfine splitting frequency of the cesium-133 atom.

frequency, velocity, action, electric charge, heat capacity, amount of substance, and luminous intensity. The specific reference quantities will be the exact values of a set of defining constants: the ground-state hyperfine splitting of the cesium-133 atom $\Delta v(^{133}\text{Cs})_{\text{hfs}}$, c, h, e, k, the Avogadro constant N_A, and the luminous efficacy K_{cd}. However, to provide continuity and ease of transition, their values will be expressed in terms of the present SI units instead of in potentially confusing new base units. Table 2 shows the new base quantities and the associated defining constants with their definitions.

Small step or giant leap?

As can be seen in tables 1 and 2, the present and future definitions of the SI have similarities, especially when one compares the present base quantities of time and length with the new base quantities of frequency and velocity. The definitions are fully equivalent, as is also the case for luminous intensity. That equivalence is because the present SI has already incorporated invariants of nature as part of its foundation, thanks to the 1967 and 1983 redefinitions of the second and meter, respectively. In fact, if the IPK were temporarily granted the status of an invariant of nature, all of the present base unit definitions could be recast into the form of the new SI.[5] After $\Delta v(^{133}\text{Cs})_{\text{hfs}}$, c, and K_{cd}, the four remaining definitions would be:

▶ The mass of the international prototype of the kilogram, $m(K)$, is exactly 1 kilogram.

▶ The magnetic permeability, μ_0, is exactly $4\pi \times 10^{-7}$ newton per ampere squared.

▶ The triple point of water, T_{TPW}, is exactly 273.16 kelvin.

▶ The molar mass of carbon-12, $M(^{12}\text{C})$, is exactly 0.012 kilogram per mole.

Because the SI has been continually evolving with new knowledge and technological advances, it might appear that the impending change is just another incremental improvement with an exchange of "invariants" in which $m(K)$, μ_0, T_{TPW}, and $M(^{12}\text{C})$

Table 1: Present SI base quantities, base units, and definitions

Base quantity	Base unit	Definition
Time	second	The second is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atom.
Length	meter	The meter is the length of the path traveled by light in vacuum during a time interval of 1/299 792 458 of a second.
Mass	kilogram	The kilogram is the unit of mass; it is equal to the mass of the international prototype of the kilogram.
Electric current	ampere	The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross section, and placed 1 meter apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per meter of length.
Thermodynamic temperature	kelvin	The kelvin, unit of thermodynamic temperature, is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water.
Amount of substance	mole	The mole is the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon-12; the elementary entities must be specified and may be atoms, molecules, ions, electrons, other particles, or specified groups of such particles.
Luminous intensity	candela	The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 540×10^{12} hertz and that has a radiant intensity in that direction of 1/683 watt per steradian.

Table 2: New SI base quantities, defining constants, and definitions

Base quantity	Defining constant	Definition
Frequency	$\Delta v(^{133}\text{Cs})_{\text{hfs}}$	The unperturbed ground-state hyperfine splitting frequency of the cesium-133 atom $\Delta v(^{133}\text{Cs})_{\text{hfs}}$ is exactly 9 192 631 770 hertz.
Velocity	c	The speed of light in vacuum c is exactly 299 792 458 meter per second.
Action	h	The Planck constant h is exactly $6.626X \times 10^{-34}$ joule second.
Electric charge	e	The elementary charge e is exactly $1.602X \times 10^{-19}$ coulomb.
Heat capacity	k	The Boltzmann constant k is exactly $1.380X \times 10^{-23}$ joule per kelvin.
Amount of substance	N_A	The Avogadro constant N_A is exactly $6.022X \times 10^{23}$ reciprocal mole.
Luminous intensity	K_{cd}	The luminous efficacy K_{cd} of monochromatic radiation of frequency 540×10^{12} hertz is exactly 683 lumen per watt.

The symbol X in the numerical values indicates additional digits to be set upon redefinition of the SI. The term "defining constant" is used in the broader sense to include invariants of nature such as the hyperfine splitting frequency of the cesium-133 atom and the luminous efficacy.

are replaced by h, e, k, and N_A. However, the change brings major advantages, the most conspicuous being the replacement of the IPK artifact, with its inherent problems of accessibility, lack of an explicit link to an invariant of nature, and questionable long-term stability.

By explicitly defining the values of a set of fundamental constants, measurements traceable to the SI will no longer be confined to a particular realization or experiment, such as a direct comparison between secondary mass standards and the IPK. The present definition of the meter as "the length of the path travelled by light in vacuum during a time interval of 1/299 792 458 of a second" demonstrates the scalability of an explicitly defined constant. At the small length scale, state-of-the-art x-ray interferometry measurements have determined the lattice spacing of isotopically enriched silicon-28 crystals[6] with an absolute uncertainty that is less than 10^{-18} m (relative uncertainty of 5 parts in 10^9). At the large length scale, the lunar laser ranging experiment can measure the Earth–Moon distance with an absolute uncertainty approaching 1 mm (relative uncertainty less than 1 part in 10^{11}) by timing the roundtrip flight of a laser pulse from Earth to the Moon and back.[7]

Both experiments depend on the fixed value of the speed of light, implicitly through the x-ray wavelength in the case of interferometry and explicitly in the case of lunar laser ranging. Neither experiment suffers from needing to relate the results to a standard at a specific length scale, as would be the case if the definition of the meter were still the distance between two notches on a platinum artifact.

The new SI will have an increased scalability and accessibility through the chosen set of constants because they appear in various research fields and physical theories, including special relativity, quantum mechanics, quantum electrodynamics, atomic physics, and condensed-matter physics. For example, given an exact value of h, mass at the 1-kg level can be measured on a watt balance with a relative uncertainty of 2 parts in 10^8 (see the box below). Or frequency measurements of atom recoil from the absorption and emission of photons can give mass at the level of 10^{-25} kg (a single atom) with a relative uncertainty of 2 parts in 10^9.

While there is great freedom in the choice of the set of fundamental constants to define a system of units, the selection of h, e, k, and N_A for the new SI was made in careful consideration of practicality,

The kilogram and the watt balance

By definition, the kilogram is the mass of the international prototype of the kilogram (IPK), making mass the last remaining base quantity of the SI that still depends on a physical artifact. The new SI will finally change that, but linking the kilogram to the defining constants in table 2 is not trivial. Based on several decades of experience measuring the Planck constant h via a watt balance,[17] NIST is constructing a fourth-generation watt balance, NIST-4, for the future dissemination of mass standards and a smooth transition to the new SI. The figure shows schematic drawings of the NIST-4 watt balance and its two operational modes. A unique feature of the device is the use of a wheel instead of a traditional beam balance.

In the first mode (panel a), a circular coil of length ℓ carrying current I in a radial magnetic field B (represented by blue arrows in panel a) experiences an electromagnetic force $F_{em} = IB\ell$ that exactly balances the weight of a test mass, mg. The current I is determined by measuring the voltage V_1 it generates across a standard resistor R (not shown) that has been calibrated by a quantum resistance standard. The voltage is measured with respect to a quantum voltage standard built into the watt balance. (Figure 2 gives a description of quantum electrical standards in use at NIST.) The absolute value of gravity at the test mass position is determined by using a combination of absolute and relative gravimeters and modeling.[18] The only unknown at this point is the electromagnetic $B\ell$ factor.

In the second mode (panel b), we measure the electromagnetic $B\ell$ factor by moving the coil vertically at velocity v in the same magnetic field B. That movement generates an electromotive force $V_2 = vB\ell$. A velocity drive on the counterweight side of the balance controls the coil velocity with feedback from three interferometers that monitor the coil motion. Panel b shows one of the three interferometers using green light. As in the first mode, the induced voltage V_2 is measured with respect to a quantum voltage standard.
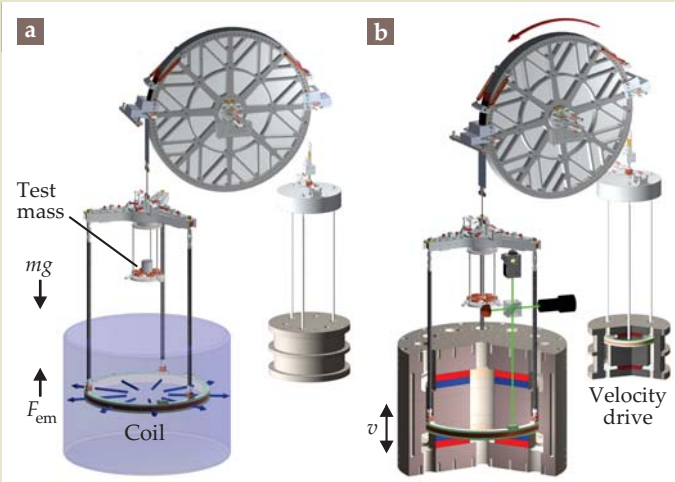
Combining the results of both modes gives

$$m = IB\ell/g = V_1 V_2/Rgv.$$

Since the electrical measurements have direct links to quantum standards, we can express them in terms of the Josephson constant $K_J = 2e/h$ and the von Klitzing constant $R_K = h/e^2$ by substituting $V_J = f/K_J$ and $R_H = R_K/i$ into the expressions for voltage and resistance, respectively. Thus a watt balance realizes the kilogram from h through the relation

$$m = h\left(\frac{in_1 n_2}{4}\right)\frac{f_1 f_2}{gv}.$$

The integer i stems from quantization in the quantum resistance standard, n_1 and n_2 are the number of Josephson junctions that are switched on in the quantum voltage standard, and f_1 and f_2 are microwave frequencies used to excite the quantum voltage standard. When completed, the NIST-4 watt balance will be able to measure mass at the 1-kg level with an absolute uncertainty in the 20-µg range.

reproducibility, accessibility, and the precision at which measurements can be made today. Comparisons of Josephson voltage standards and of quantized Hall resistance standards linked to the values of h and e (see figure 2) have relative uncertainties[8,9] of a few parts in 10^{18} and 10^{11}, respectively. Conversely, the gravitational constant G—which might seem a reasonable choice for a fundamental constant more directly linked to the traditional base mechanical units—is inherently difficult to measure (see the article by Clive Speake and Terry Quinn on page 27).

Impact and consequences

The impact of defining h, e, k, and N_A as exact will extend substantially beyond providing a basis for a system of units. Many other fundamental constants will simultaneously become exact due to the inherent relationships among them through the accepted laws of physics.

Another important consequence will be exact conversion factors, with no uncertainty, for expressing energy in units of joule, kilogram, inverse meter, hertz, kelvin, or electron volt. No longer will unit conversion cause an additional uncertainty component to appear—for example, when a researcher reports the mass of some particle in kilograms when in fact the measurement was in eV or hertz. In addition, many of the other fundamental constants will have substantially reduced uncertainties.

The International Council for Science's Committee on Data for Science and Technology (CODATA) periodically provides the scientific and technological communities with a self-consistent set of internationally recommended values for fundamental constants and conversion factors. (See the article by Peter Mohr and Barry Taylor, PHYSICS TODAY, March 2001, page 29.) Table 3, listing the uncertainties of a select group of fundamental constants in the present SI, based on the 2010 CODATA recommendations,[10] and in the new SI, shows the dramatic decrease in uncertainty of most of the constants. Due to new relevant data since the 2010 adjustment, the uncertainties are expected to further decrease for the upcoming 2014 CODATA adjustment.

When the CGPM approves the redefinition of the SI, the CODATA Task Group on Fundamental Constants will perform two special evaluations of the fundamental constants. The first will be similar to its periodic determinations, but with the goal of determining the best values of the defining constants h, e, k, and N_A. The second will be to determine the values and greatly reduced uncertainties of the remaining constants based on the newly exact defining constants.

The framework of the new SI, with exact defining constants, will have significant consequences for national metrology institutes and practical metrology. The value of T_{TPW} will not change, but a relative uncertainty component on the order of 1×10^{-6} or less will be added. The value of $M(^{12}C)$ will not change, but a relative uncertainty component on the order of 7×10^{-10} or less will be added. The value of $m(K)$ will not change, but a relative uncertainty on the order of 2×10^{-8} or less will be added. The IPK

Table 3. Changing uncertainties for fundamental constants

Quantity	Symbol	Present SI $u_r \times 10^9$	New SI $u_r \times 10^9$
International prototype of the kilogram	$m(K)$	0	44
Permeability of free space	μ_0	0	0.32
Permittivity of free space	ε_0	0	0.32
Triple point of water	T_{TPW}	0	910
Molar mass of carbon-12	$M(^{12}C)$	0	0.70
Planck constant	h	44	0
Elementary charge	e	22	0
Boltzmann constant	k	910	0
Avogadro constant	N_A	44	0
Molar gas constant	R	910	0
Faraday constant	F	22	0
Stefan–Boltzmann constant	σ	3600	0
Electron mass	m_e	44	0.64
Atomic mass unit	m_u	44	0.70
Mass of carbon-12	$m(^{12}C)$	44	0.70
Josephson constant	K_J	22	0
von Klitzing constant	R_K	0.32	0
Fine-structure constant	α	0.32	0.32
$E = mc^2$ energy equivalent	J↔kg	0	0
$E = hc/\lambda$ energy equivalent	J↔m^{-1}	44	0
$E = h\nu$ energy equivalent	J↔Hz	44	0
$E = kT$ energy equivalent	J↔K	910	0
1 J = 1 (C/e) eV energy equivalent	J↔eV	22	0

Relative uncertainties, u_r, for some fundamental constants and energy equivalents are given in parts in 10^9. Present relative uncertainties are based on the 2010 CODATA adjustment of the fundamental constants.[10] Note that u_r of $m(K)$ in the present SI is 0 only by definition. The new SI relative uncertainties assume fixed values of the Planck constant h, elementary charge e, Boltzmann constant k, and Avogadro constant N_A.

will become just another artifact with no special position in the SI. Anyone with the ability to make appropriate measurements related to the defining constants will be able to realize the kilogram.

What about the ampere?

With the discoveries of the Josephson and quantum Hall effects, it became possible to conceive of quantum electrical standards that relate electrical units to h and e through the Josephson constant, $K_J = 2e/h$, and the von Klitzing constant, $R_K = h/e^2$. Figure 2 explains the operation of two such quantum electrical standards in use at NIST. In 1990 the CIPM adopted exact values for the constants, now labeled K_{J-90} and R_{K-90}, based on the best available data.[11] Since then, almost all electrical metrology has been traceable to conventional electrical units of voltage and resistance linked to K_{J-90} and R_{K-90}. However, the present SI continues to define the ampere as the current in two infinitely long, negligibly thin wires set 1 m apart that will produce a force of 2×10^{-7} N for each meter of length. That is, for a quarter of a century, almost all electrical metrology has used a system of units that is not part of the SI. Defining h and e
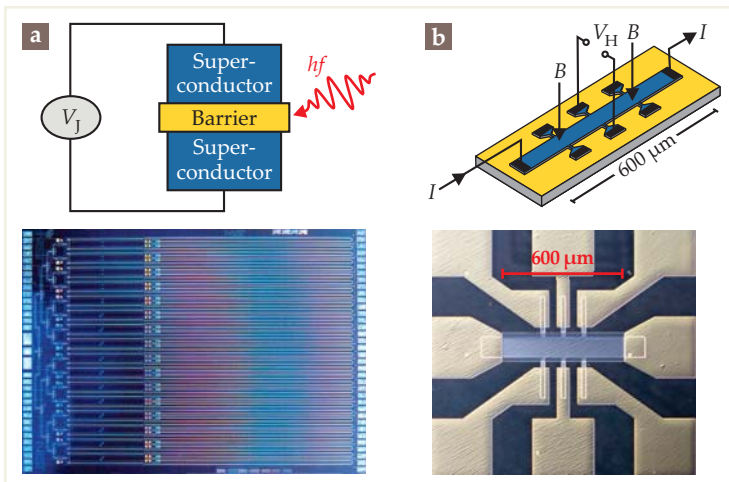
Figure 2. Quantum voltage and resistance standards. A Josephson junction **(a)** consists of two superconductors separated by a thin barrier. Microwave radiation causes a quantized DC voltage to form across the junction in steps of $V_J = f/K_J$, where f is the microwave frequency and $K_J = 2e/h$ is the Josephson constant. A quantum voltage standard is then constructed from a series array of Josephson junctions. The 17 mm × 12 mm NIST programmable Josephson voltage standard contains approximately 270 000 Josephson junctions and is capable of producing from −10 V to 10 V by selectively switching different subsets of Josephson junctions in steps as small as 484 μV. **(b)** A conducting bar with current I in an orthogonal magnetic field B develops a transverse Hall voltage V_H. For a two-dimensional bar (or, equivalently, a 2D electron gas) at low temperature and large magnetic field, the Hall voltage is quantized such that $V_H/I = R_H = R_K/i$, where $R_K = h/e^2$ is the von Klitzing constant and $i = 1, 2, 3, \ldots$. The quantized Hall resistance R_H is then used to calibrate standard resistors through precise cryogenic current comparators. Shown is a NIST graphene quantum Hall device grown from silicon carbide.

exactly will bring electrical metrology back into the SI.

Another significant impact on electrical metrology will arise from the anticipated assigned values for K_J and R_K. Taking into consideration new relevant input data for the 2014 CODATA adjustment of the fundamental constants,[12] the new fixed values of the Josephson and von Klitzing constants would lead to relative changes in voltage and resistance measurements on the order of 1×10^{-7} and 2×10^{-8}, respectively. Those changes are within the relative uncertainties of 4×10^{-7} for voltage and 1×10^{-7} for resistance measurements assigned by the CIPM's Consultative Committee for Electricity (now the Consultative Committee for Electricity and Magnetism) when translating between the 1990 conventional and SI electrical units.[13]

Even though the values of the permeability and permittivity of free space—μ_0 and ε_0, respectively—will not change upon redefinition, they will no longer be exact. There will be an additional relative uncertainty component on the order of 3×10^{-10} or less to any electrical measurement that is directly linked to μ_0 and ε_0.

Getting the word out

The idea of basing the SI entirely on invariants of nature has been simmering for at least a decade.[14]

In 2007 the 23rd CGPM recommended continuing the pursuit of relevant experiments to provide data for the fixed values of the fundamental constants, developing experimental procedures for practical realizations (mises en pratique) of the units within the new framework, and initiating awareness campaigns of a new SI based on fundamental constants.

In 2011 the 24th CGPM formally adopted a proposal from the CIPM that specified the defining constants listed in table 2 as the new SI. Since then, the international metrology community has made significant progress in providing consistent results with reduced uncertainties from the relevant experiments that determine h, e, k, and N_A. The consultative committees are completing the set of mises en pratique, and the 26th CGPM is expected to adopt the new SI in 2018.

Although word of the impending redefinition has reached a significant number of user communities and legislative bodies of metrology, the 23rd CGPM's call for public outreach needs to be pursued more assertively. What may be a boon for the scientific community could prompt utter confusion in the broader community, and it is the duty of metrologists to inform the public of the impending change in the SI. A lesson can be learned from the introduction of the metric system during the French Revolution.[15] At that time, many different systems of weights and measures were used in Paris (if one actually had any bread to weigh). Politicians imposed the new, totally foreign metric system on a starving population in what can be considered one of the greatest blunders in science communication. Widespread confusion followed, and the nascent metric system was so unpopular that in 1812 Napoleon temporarily suspended its compulsory use.

For the vast majority of the general public, the appropriate message is that when the relevant metrological organizations and committees successfully complete the redefinition, it will have no immediate noticeable effect. Bathroom and grocery scales will continue to function as before; indeed, the 1799 platinum artifact standards deposited in the Archives de la République in Paris will still be more than sufficient for most everyday weights and measures.

For teachers and others who are familiar and comfortable with the present SI and its definitions of base units, the transition may be more difficult. Physics education often starts by exploring the readily available, observable phenomena of classical mechanics followed by the introduction of classical electromagnetism before moving on to quantum mechanics and special and general relativity. With the advent of the new SI, instructors in basic classes may have to explain how a familiar quantity such as mass is directly related to the Planck constant, a subject not typically taught in introductory-level courses. One explanation is to introduce the concept of measuring mechanical power with respect to electrical power using a watt balance.[16] The box on page 38 gives an example of how such an explanation might go.

For the metrological and scientific communi-

ties, the change to the new SI will be profound. The newfound strength of the revised SI is not the ability to measure single values such as 1 kg or 1 A but the ability to scale measurements over 25 orders of magnitude using a single system of units that inherently has no uncertainty. No longer will the apparent realization of a base quantity be restricted to a single value and method.

Manufacturers will be able to use any viable method available on the factory floor to realize their measurements, at the scale needed, through exact values of the fundamental constants. For example, a pharmaceutical company might measure out doses using a microwatt balance. Such a balance could be equipped with a commercially available voltmeter and be linked to quantum electrical standards, in contrast to traditional scales that require a chain of calibrations that lead back to the IPK. Moreover, researchers will no longer have the burden of determining and reporting uncertainties associated with various energy conversion factors as a part of their results.

Employing constants of nature as the basis of our system of units for measurement will lead to finer tools for investigating the physical world, which will increase the probability of discovering new phenomena. Perhaps future research will show time variation in some of the accepted fundamental constants of nature. If so, just as the meter's basis changed from the circumference of Earth to the speed of light, the SI will incorporate the new knowledge accordingly. By continuously evolving, the SI will remain a system based on a universal measure, "for all time, for all people."
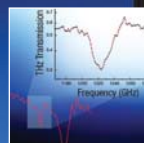
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